Beam Dynamics for Induction Accelerators

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An induction linac uses pulsed power that is applied directly, without any intervening resonant cavities, to accelerate a charged particle pulse. Relative to an rf linac this approach allows for a large beam pipe aperture capable of transporting a large current with long pulse duration. The mean accelerating gradient is expected to be relatively low (less than about 1.5 MV/m), but the potential efficiency of energy transfer is large. A multiple-beam induction linac is therefore a natural candidate accelerator for a heavy ion fusion (HIF) driver. However, the accelerated beams must meet stringent requirements on occupied phase space volume in order to be focused accurately and with small radius onto the fusion target. Dynamical considerations in the beam injector and linac, as well as in final compression, final focus and the fusion chamber, determine the quality of the driver beams as they approach the target. Requirements and tolerances derived from beam dynamics strongly influence the linac configuration and component design.

After a brief summary of dynamical considerations, two major topics are addressed here: transportable current limits, which determine the choice of focal system for the linac; and longitudinal control of the beams, which are potential destabilized by their interaction with the pulsed power system.

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1.1 Linear Induction Accelerators

A multiple-beam induction linac (LIA) is a natural candidate for a Heavy Ion Fusion driver. This type of machine has features in common with other accelerators, such as requirements for high vacuum and alignment, however there are distinctive features arising from the induction method of acceleration. We begin with a brief discussion of this interesting technology as it relates to the dynamics of a charged particle beam.

An induction linac uses a pulsed power circuit, where the pulser imposes an electric field directly on a charged particle beam at a gap in the beam transport structure. This is not a high Q cavity driven at resonance, as is the case for an rf linac. In simplified terms, charge stored in a capacitor is switched to produce a potential ΔV across the gap. A ferrite or highly laminated ferromagnetic induction core encircles the beam pipe near the gap and prevents a short to ground for a finite time Δt given by Faraday's law:

$$\Delta V = A\left(\Delta B/\Delta t\right) , \tag{1}$$

where ΔB and A are the core's average "flux swing" and longitudinal cross section area. For example $\Delta B = 2.0T$, $A = .10m^2$ and $\Delta V = 200kV$ gives $\Delta t = 1.0\mu s$ before the field collapses. Additional pulser components can shape the field in time, but in general it should be nearly constant while the beam is in the gap.

An important feature of the LIA is its potential electrical efficiency. Energy from the capacitor goes into the beam and also heats the core. Ideally there is no energy reflected from the gap back into the pulser, i.e. there is an impedance match to beam current, which must be at least 1.0kA. In general energy is lost to the circuit, but special design could capture much of the reflected energy, resulting in electrical efficiency predicted to be as high as 40% (this has not yet been attempted).

The beam's return current necessarily flows through the pulser circuit and induces a reverse field in the acceleration gap. In addition to reducing ΔV the beam's interaction with the pulser results in growing waves moving forward in the linac but backwards in the ion beam.

2.1 Beam Dynamics Overview

Since resonant cavities are not required (or desired) for acceleration in a LIA, the beam pipe and gap can have a very large aperture compared with that of an rf linac. This important feature accommodates high beam current, which in turn leads to complicated dynamics in both the transverse (x, y) and longitudinal (z) directions.

The main experience to date in design and operation of LIAs is with electron machines built primarily for defense-related purposes. Representative examples are ETA (5.0MeV, 1.0kA), ATA (50MeV, 1.0-10.0kA), and DARHT (20MeV, 1.0kA). Heavy Ion LIAs, built for Heavy Ion Fusion and Warm Dense Matter experiments, have much lower kinetic energy: MBE-4 (.4MeV, 50mA/K^+ beam) and the recently completed NDCX II (1.2MeV Li^+). The low energies of the ion machines are a consequence of their long pulse duration at low energy. Light Ion Fusion experiments used pulse power applied to one or a few gaps to produce large radius converging beams of very high current (many kilo-amperes), but these machines hd little resemblance to the conceptual HIF LIAs which are of interest here.

Although induction cores and pulsed power circuitry are very similar for electron and ion linacs, there are important differences in the beam dynamics. First, because electrons move at essentially the speed of light, they have no issues of longitudinal confinement or bunching instability. But a slow moving ion pulse must be prevented from expanding longitudinally (due to its space charge force), and a predicted bunching instability must be suppressed. The main dynamics issue for electron LIAs has been the Beam Breakup Instability (BBU), which is a growing hose-like motion of the beam driven by its interaction with the accelerating gap structure. Predictions of BBU for conceptual HIF drivers find that the growth rate is insignificant, due to the required strong focusing and large ion mass. Electron LIAs have used solenoids for transverse confinement, probably because of their relative simple construction and low fields ($B \leq 1.0T$ due to the small electron mass). HIF LIA concepts have usually adopted superconducting quadrupole magnets for transverse focusing because of their strength, good high velocity scaling and electrical efficiency. However, to date, electrostatic quadrupoles (MBE-4) and solenoids (NDCX II) have been selected for low cost and well-developed technology.

A very significant and distinct feature of HIF LIAs is the transport of multiple beams (~100) in parallel. This appears to be possible with a multiple beam quadrupole structure, but it has not been shown that parallel solenoid channels can be arranged so that they do not interfere with each other through their magnet end fringe fields.

Relatively speaking, electron beams are “hot” and ion beams are “cold”. This statement can be expressed quantitatively in terms of transverse and longitudinal emittance, which are a measure of occupied phase space area. The essential point for HIF dynamics is that, absent external focusing, an ion beam will expand from the force produced by its own space charge rather than transverse pressure. For electron beams this ordering of forces is reversed; an HIF beam (or “beamlet”) in a multi-beam structure must remain very, very cold in order to be focused to a small radius ($\approx$1.0mm) in the fusion chamber. Typically this implies that longitudinal momentum spread and angular spread are less than about 10^{-3} at the end of acceleration. The HIF beams emerge from their multi-beam injector at about 2.0MeV in a state of cold laminar flow. This condition must be maintained during acceleration despite the possibility of heating from magnet and pulser errors, electron clouds, instability, etc.

The design of a large accelerator system, e.g. HIDIX, has a formal procedure of conceptual design, scientific design, detailed design, etc. Informally, one or more rough designs must precede the formal process and all dynamical issues must be resolved at an early stage. This end-to-end treatment of dynamics must include all significant processes and be sound. A community that develops components, performs experiments, and simulates must produce the end-to-end design model. This was done effectively for NDCX II (but after the original proposal). However, no such integrated design model exists for a LIA-based driver system. Such a tool, even in preliminary form, would help guide HIF research programs in the future.

In the following section a brief discussion of the choice of focusing systems and longitudinal instability is presented. Below is a list of some dynamical considerations,

Design of transport systems with multibeam interactions - halos?
Vacuum in acceleration gaps - 10^{-8} torr good enough?

Beam loss - activation, magnet operation, e cloud
Steering - all beams separately?
Alignment - all beams separately?
Diagnostics - all beams separately?
Longitudinal control - feed forward correction?
Source reliability - for ~ 100 beams!
Extra beams for reliability
Special operations - beam bending, splitting, combining
Electrical efficiency - special pulser circuits?
Magnet aberrations - emittance growth
Transverse/longitudinal coupling-stable?
And more ……

All goes into an integrated end-to-end dynamical model.

3.1 Transverse Focusing

Three main types of externally imposed focusing elements are considered for an HIF LIA: solenoids, magnetic quadrupoles, and electrostatic quadrupoles. Other types of focusing (and defocusing) exist and are even inevitable, but they should be minimized by design in an HIF driver to maintain cold laminar flow of the beamlets. These include: electron clouds, higher order magnetic and electrostatic multipoles, weak bend focusing, beam-beam interaction in gaps, reflection from pipe walls, image charge and current, magnet fringe fields, screens and wires inside the beam, and more. Acceleration gaps produce a small net focal force that also needs to be included in a dynamical model, but it is only really important in injectors.

A simple model equation for the transverse (x) motion of an ion is:

$$\frac{d^2x}{dt^2} \approx -\omega_0^2 x + \frac{Ze}{\gamma M}\left(E_x - \upsilon B_y\right), \tag{2}$$

where ω_0 is the effective focal frequency from the quadrupoles or solenoids and (E_x, B_y) are self-generated fields of the beam. Eqn. (2) may be further approximated by

$$\frac{d^2x}{dt^2} \approx -\left(\omega_0^2 - \frac{Ze}{\gamma M}\frac{\rho}{2\varepsilon_0\gamma^2}\right)x \ , \tag{3}$$

where ρ is the mean space charge density of the beam and the factor of $1/\gamma^2$ is from partial cancellation by the beam's magnetic field. Since for a cold beam we must have a near cancellation on the rhs of eqn. (3), an approximate expression for transportable charge density is

$$\rho \approx \frac{2\varepsilon_0\gamma^3 M}{Ze}\omega_0^2. \tag{4}$$

Large ρ is a good figure of merit for a focal system, but there are additional considerations. A first point is that the phase advance per focal period (length P) is limited by a stability condition:

$$\omega_0 \rho/\upsilon \le 120^\circ \tag{5}$$

for quadrupole transport to avoid growing pulsations of the beam's radius. In practice 80° has been shown in simulations to be a safe limit that avoids emittance growth and particle halos. A similar condition applies to solenoids. Second, line charge density $\lambda = \pi a^2 \rho$ for beam radius a, is also an important figure of merit because the number of parallel beams is usually limited, say $\mathrm{N} \approx 100$ are needed for symmetrical deposition on the target. But for given ρ, the beam radius is limited by the possible field strength of the focal elements, say $|B| \le 10T$ in superconducting wire and $|E| \le 5\, MV/m$ on electrodes. Electrostatic elements are also constrained by the potential on electrical feedthroughs, 100kV being an approximate practical limit.

A solenoid transport limit is easily estimated by noting that in a frame rotating at half the cyclotron frequency, i.e. the Larmor frequency $(-\omega_c/2)$, the focusing frequency is also the Larmor frequency:

$$\omega_0^2 = \left(\frac{\omega_c}{2}\right)^2 = \left(\frac{ZeB}{2\gamma M}\right)^2, \tag{6}$$

$$\rho \approx \frac{2\varepsilon_0 \gamma^3 M}{Ze}\left(\frac{ZeB}{2\gamma M}\right)^2 = \frac{\varepsilon_0}{2}\frac{Ze\gamma B^2}{M}, \tag{7}$$

$$\lambda = \rho\pi a^2 = \left(10\frac{\mu C}{m}\right)\left(\frac{Z}{M/133amu}\right)\left(\frac{B}{10T}\right)^2\left(\frac{a}{10cm}\right)^2. \tag{8}$$

Here B^2 should be replaced by its average value – a significant reduction.

Eqn. (8) makes solenoid focusing look pretty good for large radius beams because B does not need to increase with a. For a single heavy ion beam of moderate energy (less than about 100MeV) it may be optimal, but as mentioned, there may be a problem for multiple beams.

Magnetic quadrupole transport applies strong transverse fields that alternate in sign:

$$\vec{B} = \pm B'\left(x\hat{e}_y + y\hat{e}_x\right). \tag{9}$$

A single quadrupole of transverse gradient B' focuses in one direction (e.g. $\hat{e}_x$) and defocuses in the other (e.g. $\hat{e}_y$). By alternating the polarity along the transport lattice a net focus is produced for both directions:

$$\omega_0^2 \approx \frac{\mathrm{P}^2}{96}\left(\frac{Ze}{\gamma M}\right)^2 B'^2. \tag{10}$$

Assuming 50% field occupancy and $\omega_0\, p/\upsilon < 80^\circ$. The transportable charge density is

$$\rho \approx \frac{\varepsilon_0}{48}\frac{Ze\gamma}{M}\left(\mathrm{P}^2 B'^2\right), \tag{11}$$

which looks similar to the solenoid limit, but with the crucial difference that P can become large as ion velocity increases. Field in superconducting NbTi wire may reach about 6.0T. With wire at twice the beam radius we get a technological limit on the product $B'a$:

$$2B'a \approx B_{\mathrm{wire}} \le 6.0T\,. \tag{12}$$

This allows magnets to occupy a relatively small fraction of the LIA at high energy. Beam transport channels in an array can share magnet poles, with an efficient occupancy of transverse space, making this an attractive LIA component. However, there has been no prototype development of such multi-beam magnets to date.

Transport by electrostatic quadrupoles seems similar to transport by magnetic quadrupoles. Just substitute $B' \to E'/\upsilon$ in the above formulas. Then

$$\rho \approx \frac{\varepsilon_0}{48}\frac{Ze\gamma}{M}\frac{P^2 E'^2}{\upsilon^2}\ , \tag{13}$$

again with 50% field occupancy. The new factor of $1/\upsilon^2$ shows that this technology is most effective at low kinetic energy. But when the limit on electrode potential is applied, along with the stability condition, the limit on line charge density is found to be

$$\lambda \le \frac{1}{2}\frac{\mu C}{m}\left(\frac{\Phi}{100kV}\right)\gamma, \tag{14}$$

independent of Z and M. Such a low value of λ, but with high ρ, has suggested ingenious driver configurations using thousands of parallel beams. Another possibility is to use electrostatic quadrupoles at low energy with perhaps 400 parallel beams, and make a four-to-one combining operation followed by magnetic quadrupoles at about 50MeV.

4.1 Longitudinal Stability

Unlike transverse confinement, a model for longitudinal dynamics of high current, multiple beams is not yet well developed. The main difficulty is complexity; all the beams interact with each other, both directly and via their net return current through the pulser. Also, a general model of the circuitry that includes cores and gap geometry, which is valid over a wide range if perturbation frequencies, is not yet available.

A rough 1-d model of longitudinal dynamics is given here and displays some essential features. Let I denote the total current of the entire set of N beams, λ their entire line change density, and υ their velocity. These variables are functions of time (t) and distance (z). The beams are accelerated by the net electric field E, which is also a function of t and z, and is considered to be an average over several gaps and focal sections. We then have three coupled equations for the smooth variables:

$$I = \lambda \upsilon, \tag{15}$$

$$\frac{\partial I}{\partial t} + \frac{\partial \lambda}{\partial z} = 0, \tag{16}$$

$$\frac{\partial \upsilon}{\partial t} + \upsilon \frac{\partial \upsilon}{\partial z} = \frac{Ze}{\gamma^3 M}\left(\mathrm{E}_0 + \mathrm{E}_I - g\frac{\partial \lambda}{\partial z}\right). \tag{17}$$

Here E_0 is the pulser-generated field (in the absence of a beam), and E_I is the field induced by the return current (-I). The direct space charge force appears proportional to $\partial\lambda/\partial z$, with a multiplicative factor (g) determined from the transport and gap geometry. The electric field E_I induced by the beam is approximated using a circuit equation with parallel resistance, inductance, and capacitance. Here R, L, and 1/C are smoothed values, so R has units Ohm/m, etc:

$$-I = \frac{E_I}{R} + C\frac{\partial E_I}{\partial t} + \frac{1}{L}\int^t E_I(t') \ . \tag{18}$$

A natural equilibrium is found with the net field on the rhs of eqn. (17) being is just the desired acceleration field. However, perturbations of the beam current can exhibit unstable growth. The inductive term clearly amplifies bunching while the capacitance opposes inductance and is stabilizing to an incomplete degree. The space charge term acts to propogate waves along the beam, analogous to sound, both forwards and backwards, while the resistance can cause these waves to grow in the backwards direction. A resistance without any other forces causes growth of bunches at unphysical rates.

The above analysis assumes the applicability of a very simple field model. At low frequencies (on the order of an inverse pulse duration) the circuit parameters are closely related to pulser and core properties and therefore the electrical efficiency. However, high frequencies may require different L, R, C or a different circuit model. Unstable growth distances of about 100m have previously been estimated. A feed-forward correction system may be effective in controlling these low frequency perturbations. Another method of stabilization is by momentum spread, which damps at sufficiently short wave length. Earlier estimates of required spread concluded $\Delta P/P \approx 10^{-2}$ was sufficient for complete stability with representative systems parameters, but this is clearly too much spread for final focus (after final compression amplifies it to $\Delta P/P \approx .1$ or higher). Other aspects of instability requiring renewed study are multi-beam effects and high order cavity modes.